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Design of Linear Phase Nonuniform Filter Banks with Interpolated Prototype Filters

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Abstract

Most of nonuniform filter banks designed by existing methods are not linear phase. In order to solving the problem, a novel method for designing linear-phase nonuniform filter banks is proposed in this paper. By analyzing the filter banks structure with block decimation transformed to rational decimation, we get an interpolated filter banks structure which removes the mirror frequency brought from interpolation. The filter banks are obtained by cosine modulation in this paper, and the prototype filters are interpolated and filtered before modulation. To make sure the significant aliasing distortions are cancelled completely, all prototype filters are designed with consistent transition band performance, using the optimization algorithm. Simulation results demonstrate that the linear-phase nonuniform filter banks designed by the proposed method have small amplitude distortions and aliasing distortions.

Keywords: Linear-Phase; Nonuniform Filter Banks; aliasing distortion; cosine modulation; interpolated

1. Introduction

Nonuniform filter banks (NUFBs) are widely used in many signal processing applications because of their flexibility in partitioning subbands. In some specific applications, such as image coding, it is crucial for all filters to have linear-phase (LP) property. This is LP filters can avoid artifacts in the reconstructed images. However most of NUFBs using existing design method are not LP[1-3]. A method of designing LP NUFBs is proposed in [4], where certain subbands of an LP uniform filter banks are recombined by synthesis filters of transmultiplexers, But this method has large implementation complexity and system

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delay. Uniform analysis filters and synthesis filters of transmultiplexers must have matching amplitude response for cancelling aliasing distortion. In [5]-[6], direct subband merging of cosine-modulated filter banks is studied. Because of the conflict between passband flatness and aliasing cancellation, the direct subbands merging have not LP property. Thus, in the context, it is valuable to exploit the efficient design of NUFBs with LP property.

In this paper, a novel method with cosine modulating interpolated prototype filters is presented for the design of LP NUFBs. Thanks to the interpolated filter banks structure, prototype filters can be designed using optimization algorithm with the same optimized parameters except different passband widths. Thus, all prototype filters have the same shape of transition band to cancel aliasing distortion efficiently. As illustrated by example, a LP NUFBs with simple design and excellent performance are present.

2. The structure of interpolated nonuniform filter banks

Fig.1(a) shows the M-band analysis-synthesis nonuniform filter banks system with block decimation factors p_k / q_k , $0 \le k \le N-1$, $N \le M$. We assume that factors p_k and q_k are coprime and the NUFBs are critically sampled, that is

$$\sum_{k=0}^{N-1} \frac{q_k}{p_k} = 1 . \tag{1}$$

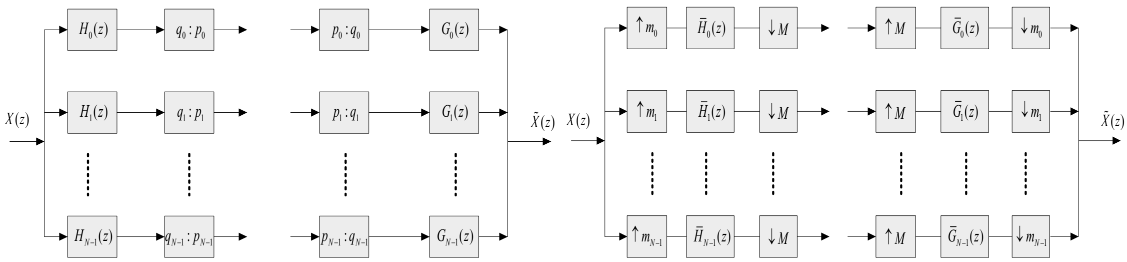


Fig.1 (a)An M-band analysis-synthesis nonuniform filter bank system with block decimation; (b)The interpolated NUFBs with rational decimation

Fig.1(b) is the equivalent form of Fig.1(a). Define least common multiple of $q_0, q_1, \ldots q_{N-1}$ as M , then $m_k = p_k \dfrac{M}{q_k}$, $0 \le k \le N-1$. In Fig.1(b), NUFBs with block decimation factors are improved to the interpolated NUFBs with rational decimation, where we need remove the mirror frequency of $P_k(z^{m_k})$ by the lowpass filter $B_k(z)$ before modulation. That is

$$\bar{P}_k(z) = P_k(z^{m_k}) B_k(z) . \tag{2}$$

NUFBs $\bar{H}_k(z)$, $\bar{G}_k(z)$ are obtained by cosine modulating prototype $\bar{P}_k(z)$.

$$\bar{H}_k(z) = H_k(z^{m_k}) B'_k(z) = e^{j\theta_k} \bar{U}_k(z) + e^{-j\theta_k} \bar{V}_k(z) \qquad \bar{G}_k(z) = G_k(z^{m_k}) B'_k(z) = e^{-j\theta_k} \bar{U}_k(z) + e^{j\theta_k} \bar{V}_k(z) \tag{3}$$

where $B'_k(z)$ are bandpass filters by shifting frequency of $B_k(z)$. θ_k are chosen to satisfy the aliasing cancellation constraint. In order to ensure the LP property of $\bar{H}_k(z)$ and $\bar{G}_k(z)$, θ_k has to be $[0, \pm\frac{\pi}{2}, \pi]$.

$$\bar{U}_k(z) = W_{2M}^{(r_k+0.5)(N_k-1)/2} \bar{P}_k(z W_{2M}^{(r_k+0.5)}) \qquad \bar{V}_k(z) = W_{2M}^{-(r_k+0.5)(N_k-1)/2} \bar{P}_k(z W_{2M}^{-(r_k+0.5)}) \tag{4}$$

where the index r_k selects where the passband is located. The input-output relationship in the z-domain is given by

$$\tilde{X}(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{p=0}^{m_k-1}\bar{H}_k(z^{\frac{1}{m_k}}W_{m_k}^{p})\bar{G}_k(z^{\frac{1}{m_k}}W_{m_k}^{p})X(z)+\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{l=1}^{M-1}\sum_{p=0}^{m_k-1}\bar{H}_k(z^{\frac{1}{m_k}}W_{m_k}^{p}W_M^{l})\bar{G}_k(z^{\frac{1}{m_k}}W_{m_k}^{p})X(zW_M^{ml}) \,. \quad (5)$$

The distortion transfer function is

$$T(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{p=0}^{m_k-1}\bar{H}_k(z^{\frac{1}{m_k}}W_{m_k}^{p})\bar{G}_k(z^{\frac{1}{m_k}}W_{m_k}^{p}) \,. \quad (6)$$

The aliasing function is

$$A(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{l=1}^{M-1}\sum_{p=0}^{m_k-1}\bar{H}_k(z^{\frac{1}{m_k}}W_{m_k}^{p}W_M^{l})\bar{G}_k(z^{\frac{1}{m_k}}W_{m_k}^{p}) \,. \quad (7)$$

Due to (3) , The distortion transfer function and the aliasing function can be written as

$$T(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{p=0}^{m_k-1}H_k(z)G_k(z)B'_k(z^{\frac{1}{m_k}}W_{m_k}^{p})^2 \quad (8)$$

$$A(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{l=1}^{M-1}\sum_{p=0}^{m_k-1}H_k(zW_M^{lm_k})G_k(z)B'_{k(}(z^{\frac{1}{m_k}}W_{m_k}^{p}W_M^{l})B'_k(z^{\frac{1}{m_k}}W_{m_k}^{p}) \,. \quad (9)$$

From (8)(9), we can see that the amplitude distortion and aliasing error are decided by the performance of $H_k(z)$ and $G_k(z)$, namely the performance of prototype filters $P_k(z)$. If (10) is satisfied and $P_k(z)$ have the same shape of transition band, the amplitude distortion introduced by the filter banks will be eliminated and the significant aliasing can be cancelled completely.

$$\left|H_k(z)G_k(z)\right|^2+\left|H_{k+1}(z)G_{k+1}(z)\right|^2=1 \,. \quad (10)$$

(10) is got from the condition of zero amplitude distortion, it can be written to (11) about $P_k(e^{j\omega})$.

$$\int_0^{\frac{\omega_{s,k}}{2}+\frac{\omega_{s,k+1}}{2}}[\left|p_k(e^{j\omega})\right|^2+\left|p_{k+1}(e^{j[\omega-(\frac{\omega_{s,k}}{2}+\frac{\omega_{s,k+1}}{2})]})\right|^2]d\omega=1 \quad (11)$$

where $\omega_{s,k}$ is the cutoff frequency of $P_k(e^{j\omega})$. If $P_k(e^{j\omega})$ and $P_{k+1}(e^{j\omega})$ have the same shape of transition band, (11) can be written as

$$\int_0^{\omega_{s,k}}[\left|p_k(e^{j\omega})\right|^2+\left|p_k(e^{j(\omega-\omega_{s,k})})\right|^2]d\omega=1 \,. \quad (12)$$

(12) is the optimization item of designing the prototype filter $P_k(e^{j\omega})$.

3. Conditions of near perfect reconstruction

The amplitude response of filters are written as

$$\left|\bar{H}_k(e^{j\omega})\right|^2=\bar{H}_k(e^{j\omega})\bar{H}_k^*(e^{j\omega})=(e^{j2\theta_k}+e^{-j2\theta_k})\bar{U}_k(e^{j\omega})\bar{V}_k(e^{j\omega})+\bar{U}_k^2(e^{j\omega})+\bar{V}_k^2(e^{j\omega}) \,. \quad (13)$$

$\bar{U}_k(e^{j\omega})$ and $\bar{V}_k(e^{j\omega})$ do not overlap significantly except when $k=0$ and $k=N-1$. θ_k have to be $[0,\pm\frac{\pi}{2},\pi]$, so $e^{j2\theta_k}+e^{-j2\theta_k}\neq 0$. Thus, the passband of the fist filter around frequencies $\omega=0$ will not be flat and the same as the last filter around frequencies $\omega=\pi$. $T(e^{j\omega})$ will create significant distortions around the $\omega=0$ and $\omega=\pi$ in $-\pi\le\omega<\pi$ too[7]. Therefore, in order to cancel amplitude distortions and make the passband flat while maintaining the LP property of individual filters, the first and last filters have to be designed separately.

From (3) we known $\bar{g}_k(n)=\bar{h}_k(L_k-1-n)$, where L_k is the order of analysis filter $\bar{h}_k(n)$, so $\bar{G}_k(z)=z^{-(L_k-1)}\bar{H}_k(z^{-1})$. The distortion transfer function (6) can be written as

$$T(z)=\sum_{k=0}^{N-1}\frac{1}{M}\frac{1}{m_k}\sum_{p=0}^{m_k-1} z^{\frac{-(L_k-1)}{m_k}} W_{m_k}^{-p(L_k-1)}\left|\bar{H}_k(z^{\frac{1}{m_k}}W_{m_k}^{p})\right|^2 . \quad (14)$$

To eliminate the phase distortion, eq (15) should be fulfilled.

$$\frac{L_k-1}{m_k}=C, 0\le k\le N-1, C\in Z . \quad (15)$$

Assuming the orders of $P_k(z)$ are constant as L, the orders of $B_k(z)$ are $L_B m_k+1$. Due to the transition band of $B_k(z)$ is very wide, L_B can be a small constant. $B_k(z)$ are all designed by the Parks-McClellan algorithm. From (2) we can get the order of $\bar{P}_k(z)$ are $(L-1)m_k+L_B m_k+1$, then $C=L-1+L_B$.

Analyzing the aliasing function(7), we can decompose $A(z)$ with $A_k^{low}(z)$ and $A_k^{high}(z)$ [8], that is

$$\begin{aligned} A_k^{low}(z)&=\frac{1}{M}[e^{-j2\theta_k}\bar{U}_k(z)\bar{V}_k(zW_M^{l_k})+e^{j2\theta_k}\bar{V}_k(z)\bar{U}_k(zW_M^{-l_k})] \\ A_k^{high}(z)&=\frac{1}{M}[e^{-j2\theta_k}\bar{U}_k(z)\bar{V}_k(zW_M^{l_k+1})+e^{j2\theta_k}\bar{V}_k(z)\bar{U}_k(zW_M^{-(l_k+1)})] \end{aligned} . \quad (16)$$

In order to cancel significant aliasing distortions, the conditions followed have to be satisfied[7-9].

$$\begin{aligned} &a)A_k^{low}(z)\downarrow m_k+A_{k+1}^{low}(z)\downarrow m_{k+1}=0 \qquad b)A_k^{low}(z)\downarrow m_k+A_{k+1}^{high}(z)\downarrow m_{k+1}=0 \\ &c)A_k^{high}(z)\downarrow m_k+A_{k+1}^{low}(z)\downarrow m_{k+1}=0 \qquad d)A_k^{high}(z)\downarrow m_k+A_{k+1}^{high}(z)\downarrow m_{k+1}=0 \end{aligned} \quad (17)$$

where $0\le k\le N-2$. In each coupling, if the magnitude responses of two aliasing terms have the same amount at the same frequency point and their phases differ by π, the significant aliasing can be cancelled completely. That requires θ_k are chosen as $[0 \text{ or } \pi]$ for symmetry and $[\frac{\pi}{2} \text{ or } -\frac{\pi}{2}]$ for antisymmetry alternately and the prototype filters $P_k(z)$ have the same shape of transition band. The findings up to now can be summarized to two conditions followed.

1) The analysis filters $\bar{h}_k(n)$, $0\le k\le N-1$, satisfy an alternate symmetry property. That is $\bar{h}_k(n)$ are symmetry and antisymmetry alternately.

2) The prototype $P_k(z)$, $1\le k\le N-2$ and $H_0(z)$, $H_{N-1}(z)$ have the same characteristic of transition band, namely the same transition bandwidth and transition band attenuation.

An efficient iterative design method for the prototype filter of uniform filter banks is proposed in [10]. Optimization includes two items as (18).

$$E_1=\int_0^{\pi/M}[\left|P(e^{j\omega})\right|^2+\left|P_k(e^{j(\omega-(\pi/M))})\right|^2-1]^2 d\omega \quad \text{and} \quad E_2=\int_{\omega_s}^{\pi}\left|P(e^{j\omega})\right|^2 d\omega \quad (18)$$

where $\omega_s=\frac{\pi}{2M}+Bg$, Bg is the required transition bandwidth. Comparing E_1 with eq(12), they have the similar form. So $P_k(z)$ can be designed using the optimization algorithm in reference [10]. Eq(18) are rewritten as

$$E_{k,1}=\int_0^{\pi m_k/M}[\left|P_k(e^{j\omega})\right|^2+\left|P_k(e^{j(\omega-(\pi m_k/M))})\right|^2-1]^2 d\omega \quad \text{and} \quad E_{k,2}=\int_{\omega_{k,s}}^{\pi}\left|P_k(e^{j\omega})\right|^2 d\omega \quad (19)$$

where $\omega_{k,s}=\frac{\pi m_k}{2M}+Bg$, $1\le k\le N-2$, Bg is a constant for all k. The bandwidth of $P_k(z)$ is $\frac{\pi m_k}{M}$, Because $P_k(z)$ have the same order and transition bandwidth, it will keep the same transition band shape after optimization. $h_0(n)$ and $h_{N-1}(n)$ are designed using the optimization algorithm too. Actually, $h_0(n)$ is a prototype filter whose bandwidth is $\frac{2\pi m_0}{M}$. $h_{N-1}(n)$ is obtained by frequency shifting π of a prototype filter $h_{N-1}^{low}(n)$, that is $h_{N-1}(n)=(-1)^n h_{N-1}^{low}(n)$, where the bandwidth of $h_{N-1}^{low}(n)$ is $\frac{2\pi m_{N-1}}{M}$. So $h_0(n)$ is symmetry and $h_{N-1}(n)$ is antisymmetry. When designing $h_0(n)$ or $h_{N-1}^{low}(n)$, we change the m_k in (19) into $2m_0$ or $2m_{N-1}$. Bg is not changed. The orders of $H_0(z)$ and $H_{N-1}^{low}(z)$ are L as the same as $P_k(z)$. So $H_0(z)$, $H_{N-1}^{low}(z)$ and $P_k(z)$ have the same shape of transition bands.

4. Simulation results

The reconstruction performance of NUFBs is decided by the flatness of passband, attenuation of stopband and consistency of transition band of individual filter. In this paper, we design the prototype filters of NUFBs with optimization algorithm to get more excellent reconstruction performance.

The systematic design procedures of the proposed LP NUFBs are summarized below. $P_k(z)$, $1 \le k \le N-2$, $H_0(z)$, $H_{N-1}^{low}(z)$ are all designed with the optimization algorithm in [10]. $B_k(z)$, $0 \le k \le N-1$ are all design by Parks-McClellan algorithm.

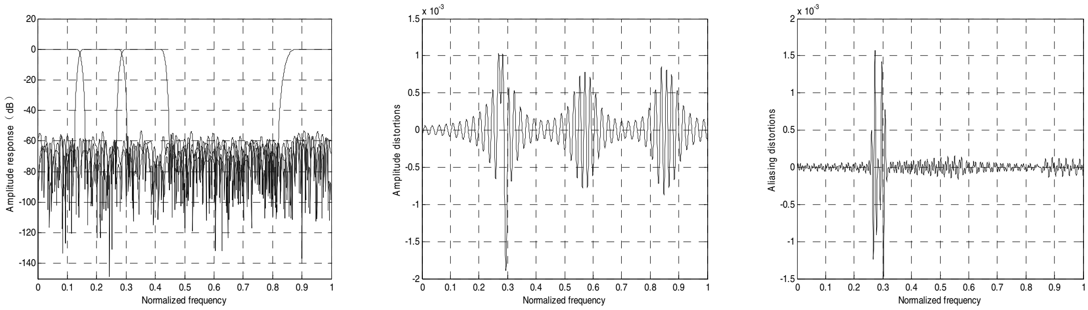
1) Choose the filter order L , L_B and constant transition width Bg . Then the order of $P_k(z^{m_k})$ are $(L-1)m_k+1$; the order of $B_k(z)$ are $L_B m_k+1$; the order of $\bar{P}_k(z)$ are $(L-1)m_k + L_B m_k +1$ and $C = L-1+L_B$.

2) Design $\bar{H}_0(z)$ and $\bar{H}_{N-1}(z)$. Design $h_0(n)$ and $h_{N-1}^{low}(n)$ first. Bandwidths of $h_0(n)$ and $h_{N-1}^{low}(n)$ are respectively $\frac{2\pi m_0}{M}$ and $\frac{2\pi m_{N-1}}{M}$; the orders are L ; transition bandwidths are all Bg . Then $h_{N-1}(n)=(-1)^n h_{N-1}^{low}(n)$, and $H_0(z)$ and $H_{N-1}(z)$ are interpolated by m_0 and m_k respectively. Thus, $\bar{H}_0(z)=H_0(z^{m_0})B_0(z)$ and $\bar{H}_{N-1}(z)=H_{N-1}(z^{m_{N-1}})B_{N-1}(z)$, where $B_0(z)$ is lowpass and $B_{N-1}(z)$ is highpass.

3) Design the prototype filters $P_k(z)$, $1 \le k \le N-2$. Bandwidths are $\frac{\pi m_k}{M}$; transition bandwidths are Bg ; the orders are L . $P_k(z)$ are interpolated by the corresponding m_k respectively. So $\bar{P}_k(z)=P_k(z^{m_k})B_k(z)$,where $B_k(z)$ are lowpass.

4) Cosine modulate $\bar{P}_k(z)$ to obtain $\bar{H}_k(z)$, $1 \le k \le N-2$. θ_k are chosen as $[0 \text{ or } \pi]$ for symmetry and $[\frac{\pi}{2} \text{ or } -\frac{\pi}{2}]$ for antisymmetry alternately. Frequency shifting factors $r_k = \frac{1}{m_k}\sum_{i=0}^{k-1} m_i$.

Example: In this example, a $N=4$ channels LP NUFBs is designed. Sample factors are $[\frac{2}{7},\frac{2}{7},\frac{2}{7},\frac{1}{7}]$. The parameters $L=128$; $L_B=64$; $Bg=\frac{0.4\pi}{2*7}$; frequency shifting factors $r_1=1, r_2=2$; phase shifting factors $\theta_1=\pi/2,\ \theta_2=0$. Fig.2(a) shows amplitude response of 4 channels LP NUFBs. The stopband attenuation is close to $-60dB$. Fig.2(b) shows the amplitude distortions, that is under 2×10^{-3} . Fig.2(c) shows the aliasing distortions, that is under 1.5×10^{-3} . From what the Fig.2 show, we can see the LP NUFBs have excellent reconstruction performance using the proposed method. The aliasing error and amplitude distortion are much smaller than that of [4]. The transition bandwidth is much smaller than that of [7]. Because the NUFBs are obtained by modulating several prototype filters, the proposed method has lower implementation complexity than the direct design method.



(a) (b) (c)

Fig.2 (a)Amplitude responses of analysis filters; (b)Amplitude distortions of the NUFBs; (c)Aliasing distortions of the NUFBs

5. Conclusion

In this paper, a design of cosine modulation LP NUFBs with interpolated prototype filters is proposed. The prototype filters have the same transition bandwidth and order, then they have consistent transition bands performance after optimization, and the aliasing can be cancelled completely. Besides, optimization complexity is lower thanks to filters interpolated. When the NUFBs have great channels and some of them have the same sampling factors, the proposed method is more suitable.

References

[1] J.Kovacevic and M.Vetterli, perfect reconstruction filter banks with rational sampling factors, IEEE Trans SP. Vol. 41, no.6, pp 2047-2066, June 1993

[2] P.B. Batalheiro and M.R. Petraglia; Nonuniform Subband Adaptive Filtering With Critical Sampling, IEEE Transactions on Signal Processing, vol.56,no 2, pp 565-575 Feb. 2008

[3] S.C.Chan, X.M.Xie, T.I.Yuk, Theory and design of a class of cosine modulated nonuniform filter banks, in Proc. IEEE ICASSP, vol.1, pp 504-507, May 2000

[4] X.M.Xie, S.C.chan, and T.I.Yuk. Design of linear-phase recombination nonuiform filter banks, IEEE Trans. Signal Process. Vol.54, no.7, pp 2809-2814, Jul 2006

[5] J.J.Lee and B.G.Lee, A design of nonuniform cos modulated filter banks, IEEE Trans Circuits Syst Π , vol.42, no.11, pp 732-737, Nov 1995

[6] O.Aniamut and R. Heusdens, Subband merging in cosine-modulated fliter banks, IEEE Signal Process Lett, vol 10, no.4, pp 111-114, Apr 2003

[7] X.Y.Chen, X.M.Xie, G.M.Shi and W.Zhong, Design of Linear-Phase Nonuniform Filter Banks With Partial Cosine Modulation, IEEE Trans. Signal Process. Vol.58, no.6, June 2010

[8] Fabrizio Argenti, Benedetto Brogelli, and Enrico Del Re, Design of pseudo-QMF banks with rational sampling factors using several prototype filters, IEEE Trans. Signal Process. Vol.46, no.6,pp 1709-1715 June 1998

[9] Fabrizio Argenti, Benedetto Brogelli, and Enrico Del Re, Cosine-modulated non-uniform filter banks, IEEE Digital Signal Processing Workshop Proceedings, pp 129-132, Sept. 1996

[10] Hua Xu, Wu-Sheng Lu, Antoniou, A. Efficient iterative design method for cosine-modulated QMF banks, IEEE Trans. Signal Process. Vol.44, no.7,pp 1657-1668 Jul 1996